SAT Maths lvl 2.

Real numbers

Reflexive: $x = x$ $2 = 2$

Symmetric: if $a = b$ then $b = a$

Transitive: if $a = b$ and $b = c$ then $a = c$

Addition:

- Closure: $a + b$ is a unique real number
- Commutative: $a + b = b + a$
- Associative: $(a+b)+c = a+(b+c)$
- Identity: $a + 0 = a$
- Additive inverse: $a + -a = 0$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Discriminant:
if $b^2 - 4ac$ is a square, then two rational roots.

Inequalities:

- conjunction $a < x < b$
- disjunction $x < a$ or $x > b$

Systems of equation

- contradiction = no solution
- identity = infinitely many solutions

Solid geometry + coordinates

- area of eq triangle $= s^2\sqrt{\frac{3}{4}}$
- regular polygon $A = \frac{1}{2}asn$
  where a is length of apothem
- Heron's formula: area of triangle
  $A = \sqrt{s(s-a)(s-b)(s-c)}$
  where s = semiperimeter

- Pyramid: $V = \frac{1}{3}BH$ $S = \frac{1}{2}lP + B$ (l: slant length, P: perimeter of base)
- Standard form for line equation
  $Ax + By = C$. $m = -\frac{A}{B}$
- Point-slope form
  $y - y_1 = m(x - x_1)$

Circles:

circle with centre (h, k) and radius r

$$(x-h)^2 + (y-k)^2 = r^2$$

Parabolas:

- Generally $y = ax^2 + bx + c$

  or $y = a(x-h)^2 + k$

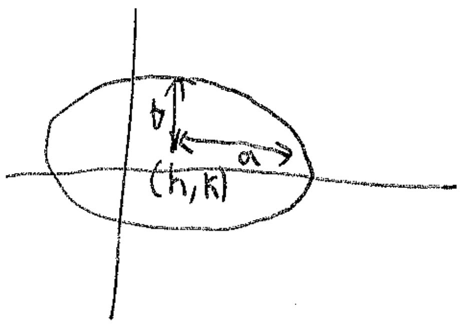
  ($\cancel{\text{centre}}$ vertex (h, k))

- Can also be

  $x = a(y-h)^2 + k$

Ellipses:

- set of points whose distances from two foci are constant.



$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad \text{(if major axis is horizontal)}$$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1 \quad \text{(if major axis vertical)}$$

- c is the distance from each focus to the centre.

$$c^2 = a^2 - b^2$$

- Eccentricity of an ellipse is less than one.

$$e = \frac{c}{a}$$

as $e \to 0$, more circular. as $e \to 1$, more stretched.

Hyperbola

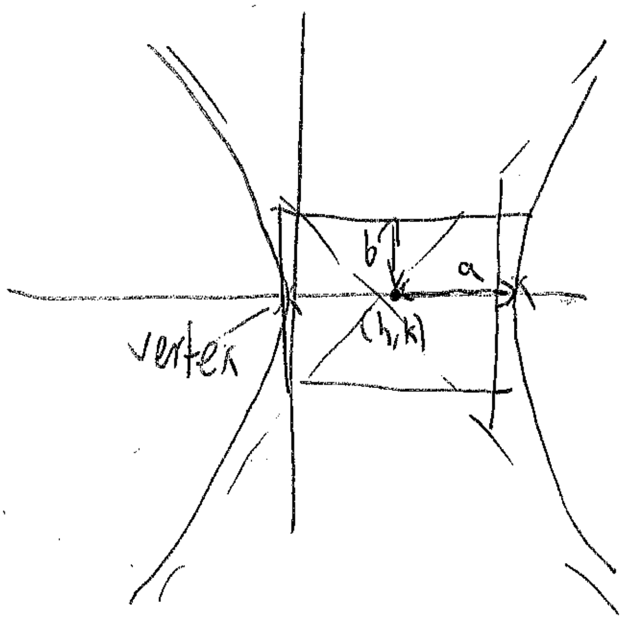
- set of points, difference of whose distances from 2 foci is constant.

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \quad (\text{for})$$

or

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1 \quad (\text{for})$$

$\rightarrow$



- $2a$ is the length of the transverse axis (connecting vertices)
- $2b$ is the length of the transverse axis

  height of an imaginary rectangle whose corners are on the asymptotes.
- $c^2 = a^2 + b^2$

$$e = \frac{c}{a}$$

Equations of asymptotes:

$$y = k \pm \frac{b}{a}(x-h) \quad \text{(for)}$$

$$y = k \pm \frac{a}{b}(x-h) \quad \text{(for)}$$

Polar coordinates

(r, θ). To convert to rectangular:

$x = r\cos\theta \qquad y = r\sin\theta \qquad x^2 + y^2 = r^2$

Trig

$\sin A = \cos(90 - A) \qquad \tan A = \cot(90 - A) \qquad \sec A = \operatorname{cosec}(90 - A)$

$\sin 2\theta = 2\sin\theta\cos\theta \qquad \cos 2\theta = \cos^2\theta - \sin^2\theta$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \cancel{2}\tan^2\theta}$$

Functions

- Identity function - $y = x$
- periodic function: $f(x) = f(x + c)$

Polynomials

Division algorithm states that $f(x) = d(x)\,q(x) + r(x)$ (divisor, quotient, remainder)

- Remainder theorem: when $f(x)$ divided by $x-r$, remainder $= f(r)$
- Factor theorem: $x-k$ is a factor of $f(x)$ if $f(k)=0$
- Descartes Rule of Signs:
  - number of positive zeroes of $f(x)$ = the number of variations in sign, or an even integer less.
  - number of negative zeroes = number of variations in sign of $f(-x)$, or an even integer less
- Rational root test:
  If a polynomial has integer coefficients, every rational zero has the form $\frac{p}{q}$ (simplified)
  where $p =$ a factor of the constant, $q =$ a factor of a_n (leading term)
- Complex zeroes occur in conjugate pairs.
  - if a root is $a+bi$, then $a-bi$ is also a zero.

Absolute value of complex number $a+bi = \sqrt{a^2+b^2}$

Logic:

Conditional: if p then q T
Converse: if q then p F
Inverse: if not p then not q F
Contrapositive: if not q then not p T

Matrices:

Determinant of $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \det X = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad-bc$

Vectors:

length = norm $= \|V\| = \sqrt{V_x^2+V_y^2}$ for $\binom{x}{y}$