10. Conservation of Momentum

10.1 Analytical solutions are often hard to come by, but it helps if we know some general principles.

N3 states that $F_{BA} = -F_{BA}$.

$$\Rightarrow \frac{dp_1}{dt} = -\frac{dp_2}{dt}$$

$$\Rightarrow \frac{d(p_1 + p_2)}{dt} = 0$$

Hence, N3 implies conservation of momentum: when two bodies interact, $m_1v_1 + m_2v_2$ is constant.

This can be generalised to the isolated interaction of many particles: $\sum_i m_i v_i = \text{constant}$.

Later it will be shown that $\sum F$ on an object is equal to $\frac{d}{dt}(p_{\text{total}})$.

Galilean relativity

Another implication of N2 (to be proved later) is the principle of Galilean relativity - the laws of physics look the same at any constant velocity.

It turns out that, from this principle, one can infer the conservation of momentum.

10.3 If we apply a small "push" between two blocks of equal mass on a frictionless surface, symmetry means that the objects will move away with a speed $v$.

If these were to bounce back from bumpers, symmetry would mean that the blocks end up at rest.

However, if the observer now moves with velocity $-v$, it will look like this: before $\frac{2v}{m}$, after $\frac{0}{m}$, $mv$ is constant.
We can extend this. Suppose the two masses have velocities \( v_1 \) and \( v_2 \), and we consider the perspective of an observer with velocity \( v_0 \).

**View from lab**

\[
\begin{align*}
\text{before} & \quad v_1 \rightarrow v_2 \\
\text{after} & \quad v' \rightarrow v''
\end{align*}
\]

**View from moving observer**

\[
\begin{align*}
\text{before} & \quad v \rightarrow \left( \frac{v_1 - v_0}{1 - \frac{v_0 v}{c^2}} \right) \\
\text{after} & \quad \left( \frac{v_1 - v_0}{1 - \frac{v_0 v}{c^2}} \right) \rightarrow \left( \frac{v_1 + v_0}{1 + \frac{v_0 v}{c^2}} \right)
\end{align*}
\]

Then we get

\[
\begin{align*}
v &= \frac{1}{2} (v_1 - v_2) + v_2 = \frac{1}{2} (v_1 + v_2) \\
\Rightarrow \quad m v_1 + m v_2 &= 2 m \cdot \frac{1}{2} (v_1 + v_2) \Rightarrow \text{momentum conserved.}
\end{align*}
\]

If we have different masses, we can just pretend that the larger mass is made of a number of smaller masses very near each other, then we apply the rules for equal masses:

\[
\begin{align*}
\begin{array}{c|c}
\text{m} & \text{2m} \\
\hline
\text{m/m} & \text{m/m}
\end{array}
\end{align*}
\]

10.4 **Momentum and energy**

The previous examples have been inelastic, KE has not been conserved, and has instead been converted to heat. Collisions between objects with no 'internal parts' are likely to be elastic, conserving KE.

10.5 **Relativistic momentum**

We must include \( Y \), where \( v^2 = v_x^2 + v_y^2 + v_z^2 \).

Because of \( c \), transfer of momentum isn't instantaneous. To satisfy conservation, we say that the EM field has momentum.

E.g. in QM, the momentum of waves is found by the de Broglie equation: \( p = \hbar / \lambda \).

Even though Newtonian mechanics has been superseded, the conservation of momentum is a deep and fundamental principle.